Blind Decision Feedback Equalizer for 64-QAM Systems

Vladimir R. Krstić[1], Miroslav L. Dukić[2]

Abstract – **This paper addresses the structure-criterion switching stability of self-adaptive decision feedback equalizer (DFE) proposed for 64-QAM systems. In the case of 64-QAM signal, this blind equalizer scheme shows an instability when its recursive part controlled by entropy-based algorithm switches operation from blind to decision-directed mode. Also, closely related to this difficulty, the problem of a robust decision-directed carrier phase recovery becomes more demanding. To eliminate these two sources of instability, the existing Soft-DFE scheme is improved by both the tap-leakage decorrelator and reduced constellation phase-locked loop.**

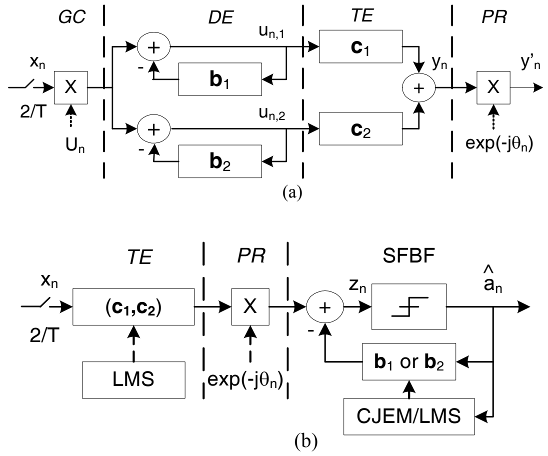
Keywords – **Blind decision feedback equalizer, joint entropy maximization criterion, reduced constellation phase-locked loop.**

I. INTRODUCTION

A blind equalization is the challenging approach of establishing a synchronous operation between transmitter and receiver terminals without relying on a referent training signal. It gives, in general, possibilities to improve the effective throughputs in a number of communication services matured on the radio and multi-point transmission systems.

As it is known, classical decision feedback equalizers (DFEs) for correct decisions show superior steady-state performances compared to linear transversal ones. At the same time, these DFEs suffer to converge in a blind scenario due to error propagation effects. To evade this phenomenon Labat *et al.* [1] designed the effective self-adaptive DFE scheme adapting both the structure and the optimization criterion. Recently, employing this concept, the new Soft-DFE scheme has been proposed in [2]. It is characterized by the entropy-based feedback filter providing soft switching from the blind to tracking operation mode. On the other hand, in [3] a similar effect was also obtained by using the predictive DFE scheme with unchangeable structure. In this case, besides adaptive feedforward and feedback filters, the DFE is also equipped with an adaptive decision device which effectively transforms its recursive part in a smooth manner.

This paper proposes the improved Soft-DFE scheme for 64-QAM single-carrier systems characterized by deep frequency-selective fading channels in the presence of a carrier frequency offset. Based on a number of computer simulations, it was shown in [4] that an instability of Soft-DFE for m-QAM (m>32) signals primarily comes from an unrestrained growth of coefficient modules of its recursive part during the blind operation mode. Besides that, the stability of the Soft-DFE is additionally imperilled by the constellation spin effects coming from the data-directed phase-locked loop (PLL) system [5]. Aiming at improving the structure-criterion switching performance of the Soft-DFE scheme in the environment of 64-QAM signal we have stabilized the whitening of received signal by the new tap-leakage decorrelator [4], and eliminated PLL instability by using the PLL with reduced signal constellation (RC-PLL).



(a)

(b)

Fig. 1. Soft-DFE scheme in (a) blind acquisition and (b) soft transition/tracking modes

The rest of the paper is organized as follows: In Section II the basic features of the original Soft-DFE scheme are presented. The innovations for 64-QAM signal are given in Section III and the results of computer simulations are presented in Section IV.

II. SOFT-DFE RECALLS

The Soft-DFE [2] is based on the self-adaptive structure consisting of the linear feedforward and nonlinear feedback equalizers defined by coefficient vectors $\{\mathbf{c}_1, \mathbf{c}_2\}$ and $\{\mathbf{b}_1, \mathbf{b}_2\}$, respectively, Fig. 1. The feedforward part of Soft-DFE is realized as a T/2 fractionally-spaced equalizer (T/2-FSE, T is the symbol period). The equalization process of Soft-DFE scheme is characterized by three operation modes which succeed each other as follows: blind acquisition, soft transition and tacking. To control structure-criterion switching, the performance index monitor is introduced which estimates the output mean square error (MSE) through the whole process of equalization.

A. Blind Acquisition Mode

In the blind acquisition mode, the Soft-DFE effectively acts as a linear equalizer (LE) including four adaptive signal transformers in the cascade as it is depicted in Fig. 1(a): gain control (*GC*), decorrelator (*DE*), FSE equalizer (*TE*) and

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phase rotator (*PR*). In the given cascaded structure a difficult task of blind equalization is divided into several easier subtasks progressing “from easier to more difficult”. In that sense it should be emphasized that *DE* and *TE* perform the most critical subtasks, while the situation with respect to *PR* is quite demanding due to an inadequate (poor) quality of symbol estimates y_n at the output of T/2-FSE controlled by the constant modulus algorithm (CMA) [6].

Formally, the pure recursive deccorrelator *DE* was obtained as a result of linear modification of the entropy-based soft decision feedback equalizer (denoted as SFBF in Fig. 1(b)) controlled by the joint entropy maximization (JEM) criterion $J_{EM}(\mathbf{b}_n) = E\{\ln|\partial r_n / \partial z_n|\}$ [7] where n is the time index ($t = nT$). This criterion, which is the real function of complex-valued arguments, transforms (iteratively) generally unknown PDF of an input sequence z_n into a sequence $r_n = g(z_n)$ succeeding the PDF of maximal Shannon’s entropy; the employed function of transformation is the complex nonlinearity $g(z) = z(1+\beta|z|^2)$ where a parameter β defines the roundness (slope) of mapping surface. For the criterion $J_{EM}(\mathbf{b}_n)$, the corresponding complex stochastic gradient algorithm CJEM-W is given by

$$\mathbf{b}_{i,n+1} = \mathbf{b}_{i,n} - \mu_W u_{i,n}\left(1-\beta_W\left|u_{i,n}\right|^2\right)\mathbf{u}_{i,n}^*, \quad i = 1,2 \tag{1}$$

where $\mathbf{b}_{i,n}$ and $\mathbf{u}_{i,n}$ are complex vectors of length N, μ_W is a step size, and β_W a signal dependent parameter.

The adaptation of *TE* is controlled by the CMA given by

$$\mathbf{c}_{i,n+1} = \mathbf{c}_{i,n} - \mu_G y_n\left(\left|y_n\right|^2 - R_2\right)\mathbf{u}_{i,n}^* \tag{2}$$

where $\mathbf{c}_{i,n}$ is a complex vector of length L, μ_G is a step size and R_2 is a signal dependent constant. The normalized statistical constant R_2 for 64-QAM signal results in $R_2 = 1.381$ for equally probable data symbols.

B. Soft Transition Mode

In the soft transition mode, one of the two *DE* whiteners - selected according to energy criterion - becomes the SFBF keeping on the JEM adaptation, while the equalizer *TE* switches the algorithm from CMA to conventional decision-directed LMS, Fig. 1(b). Formally, in this phase of adaptation, the equalizer combines the two optimization criteria: 1) the MSE criterion $J_{MSE}(\mathbf{c}_n, \theta_n)$ which jointly estimates both the coefficient vector $\mathbf{c}_n$ and the carrier phase θ_n to respond to a minimum output MSE, and 2) the JEM criterion $J_{EM}(\mathbf{b}_n)$ which estimates vector $\mathbf{b}_n$ to respond to a maximum entropy sequence at the output of SFBF. The corresponding stochastic gradient recursions which update the equalizer’s adjustable parameters $\mathbf{c}_n$, $\mathbf{b}_n$ and $\hat{\theta}_n$ during the soft transition mode are given, respectively, as follows

$$\mathbf{c}_{i,n+1} = \mathbf{c}_{i,n} - \mu_c\left(z_n - \hat{a}_n\right)e^{j\hat{\theta}_n}\mathbf{x}_{i,n}^* \tag{3}$$

$$\mathbf{b}_{n+1} = \mathbf{b}_n - \mu_D z_n\left(1-\beta_D\left|z_n\right|^2\right)\hat{\mathbf{a}}_n^* \tag{4}$$

$$\hat{\theta}_{n+1} = \hat{\theta}_n + \mu_\theta \frac{\operatorname{Im}\left(z_n \hat{a}_n^*\right)}{E\left\{\left|a_n\right|^2\right\}} \tag{5}$$

where μ_c, μ_D and μ_θ are corresponding steps, and β_D is a signal dependent constant. For the reason of simplicity, the PLL recursion (5) is given in the first order form.

C. Tracking Mode

Finally, in the tracking mode, the Soft-DFE operates as the classical DFE controlled by the well-known MSE criterion $J_{MSE}(\mathbf{c}, \mathbf{b}, \theta) = E\left\{\left|z_n - \hat{a}_n\right|^2\right\}$. The Soft-DFE switches from blind to soft transition and from soft transition to tracking mode at the threshold levels M_{TL-1} and M_{TL-2}, respectively (see Fig. 3).

III. IMPROVEMENTS FOR 64-QAM

The two dominant sources of instability of the Soft-DFE with 64-QAM are: 1) the instability caused by the unrestrained growth of *DE*’s coefficient modules, and 2) the PLL spinning induced by both a weak likelihood of estimated symbols and the crowded signal constellations of 64-QAM. To eliminate these drawbacks we have improved the Soft-DFE scheme by introducing the following innovations: 1) a tap-leakage term into the original CJEM-W recursion and 2) a reduced 64-QAM signal constellation into the commonly used decision-directed second-order PLL.

A. Tap-Leakage Algorithm for Decorrelator

Let us remind of the tap-leakage LMS (LMS-TL) algorithm [8] which was originally derived with the aim to assure a stable steady-state operation of the FSE implemented by the DSP fixed-point arithmetic. In this case, the MSE criterion is extended to $J_{MSE-TL} = J_{MSE} + \gamma\sum_{i=0}^{L-1}|c_i|^2$ where a small positive number γ controls coefficients leakage. The term $\gamma\sum_{i=0}^{L-1}|c_i|^2$ is the result of adding a zero-mean virtual noise at the input of FSE which is statistically independent of the input signal.

Now, let us apply the above tap-leakage method to our entropy-based decorrelator *DE*. From the theoretical point of view, the JEM criterion $J_{EM}(\mathbf{b}_n)$ was derived for the noiseless model of the SFBF equalizer where the correct decisions $\hat{a}_n = a_n$ were assumed. Hence, the feeding of its feedback transversal filter with a virtual noise has no sense. On the other hand, the entropy-based decorrelator *DE* was derived as a linear extension of SFBF, and it acts independently of other linear transformers in the LE cascade (except of the gain control *GS*, see Fig. 1(a)). In this situation it is reasonable to modify the criterion $J_{EM}(\mathbf{b}_n)$ locally by adding a particular noise-dependent term that directly controls *DE*'s coefficient modules and does not participate in any kind of entropy-based manipulations. Formally, the original cost function $J_{EM}(\mathbf{b}_n)$ can be modified in the following way

$$J'_{EM} = \gamma \sum_{j=1}^{N} \left|b_j\right|^2 + J_{EM} \qquad (6)$$

where the term $\gamma \sum_{j=1}^{N} \left|b_j\right|^2$ directly controls the growth of coefficient modules of the *DE* during the blind acquisition. Consequently, by repeating the same stochastic gradient method of optimization as in the case of the CJEM-W algorithm in (1), the corresponding tap-leakage variant (CJEM-TL) is obtained as follows

$$\mathbf{b}_{i,n+1} = \mathbf{b}_{i,n}(1-\gamma) - \mu_W u_n \left(1 - \beta_W \left|u_n\right|^2\right) \mathbf{u}^*_{i,n}, \quad i = 1,2 \qquad (7)$$

where γ denotes a leakage factor. In the above recursion the term $\gamma b_{j,n}$ systematically decreases the coefficient modules of *DE*.

B. Phase-Locked Loop with Reduced Constellation

In the improved RC-PLL scheme a current phase error is calculated with respect to the reduced 64-QAM constellation including only four corner symbols with the largest energy in the 64-QAM signal space. Practically, the RC-PLL performs a phase discrimination only for symbol estimates $s_n = \{y'_n, z_n\}$ (see Fig.1) satisfying inequality $\left|s_n\right|^2 \geq 90$, and for $\left|s_n\right|^2 < 90$ the phase discriminator output is set to zero; this constant radius of $\sqrt{90}$ for the 64-QAM is decided experimentally. By introducing the RC-PLL into the Soft-DFE, in this paper the dilemma is stated again – either to activate the RC-PLL from the beginning of equalization or later, when a quality of symbol estimates becomes better.

IV. COMPUTER SIMULATIONS

The equalization characteristics of the innovated Soft-DFE are analyzed and evaluated through several tests. In the first test, the MSE convergence characteristics of the Soft-DFE for both the {*DE*, CJEM-W} and the {*DE*, CJEM-TL}, are analyzed. Then, the phase error convergence is presented for the two strategies of carrier phase estimation. Finally, the effective performance features of Soft-DFE are given in the term of equalization success index (ESI). This term defines a ratio between the numbers of successfully completed equalizations and the total number of Monte Carlo runs.

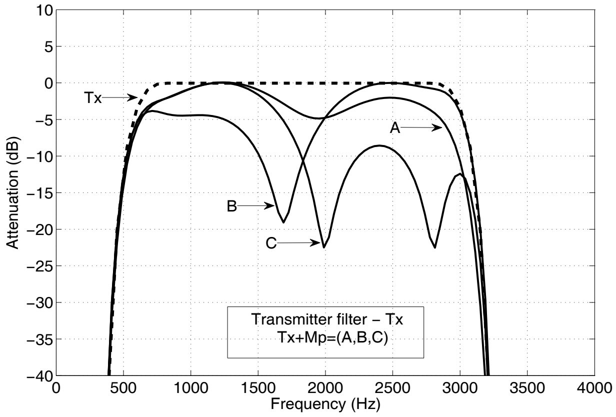

Fig. 2. Attenuation of multipath channels Mp-(A, C, E)

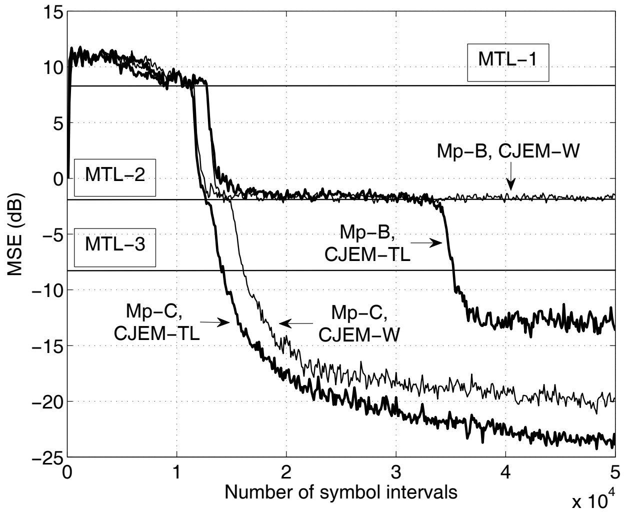

Fig. 3. One run of MSE convergence for Mp-{C, E} channels: Soft-DFE with CJEM-W and CJEM-TL algorithms.

The simulations are carried out for the time-invariant frequency-selective fading channels under a signal-to-noise ratio of 30 dB and a frequency offset uniformly distributed in the range of (0.0-0.24) Hz. Fig. 2 depicts the attenuation characteristics of three-ray channel model involved into the transmitter filter of system. The length of equalizer is L=25 and N=5 in its *TE* and *DE* parts, respectively. The initial vectors of *TE* are with zero components except the centred (reference) ones $c_{1,r} = c_{2,r} = 1$. The signal dependent parameters $\beta = \{\beta_W, \beta_D\}$ of CJEM algorithms for 64-QAM are selected to be $\beta_W = 0.9$ and $\beta_D = 2.1$, while the leakage factor optimal in the sense of maximal ESI is $\gamma = 2^{-12}$ [5]. For 64-QAM signal, the suitable selected threshold levels are M_{TL-1}=7.9 dB, M_{TL-2}=-2.2 dB and M_{TL-3}=-7.8 dB; the level MSE< M_{TL-3} signifies an equalization success, (see Fig. 3).

A. MSE Convergence: CJEM-TL Algorithm Effects

The influence of innovated CJEM-TL algorithm on the MSE convergence characteristics is illustrated in Fig. 3; the

{*DE*, CJEM-W} and {*DE*, CJEM-TL} variants of *DE* are compared for the severe channels Mp-(B, C). As can be seen, during the first two modes of operations up to the moment when MSE reaches the threshold M_{TL2}, the two curves of convergence corresponding to both {*DE*, CJEM-W} and {*DE*, CJEM-TL} practically overlap each other, but after switching the Soft-DFE to the tracking mode a divergence between these curves is apparent. Particularly, this divergence is more radical in the case of Mp-B channel where the Soft-DFE with {*DE*, JEM-W} demonstrates an equalization failure. On the other hand, in the case of {*DE*, CJEM-TL}, the convergence characteristics become more desirable. For the Mp-C channel, the Soft-DFE with CJEM-TL performs a soft switching from the blind to tracking mode so that benefits are evident: the speed of convergence is not changed after switching and a residual MSE is lower in comparison to the respective behaviour of Soft-DFE with CJEM-W. Finally, the same situation for the Mp-B channel is worthy of attention since the presence of local minima is obvious. In this case, the CJEM-TL algorithm shows ability to "attenuate" the effects of local minima and to prompt the coefficient vectors of both *TE* and SFBF equalizers to "escape" from undesirable local minima.

B. Effects of RC-PLL

Fig. 4 presents the phase error convergence characteristics of the RC-PLL for two scenarios: 1) RC-PLL-ON is turned on at the beginning of equalization and 2) RC-PLL-OFF is turned on at the start of soft transition mode. As can be seen, the phase error converges faster for the first scenario indicating that the information content of the reduced signal constellation still guaranties an acceptable quality of carrier phase estimates. It means that there is no reason to waste a time of nearly 10000 iterations during the blind mode and then to turn on the PLL. For both scenarios, the PLL switches itself from reduced to full constellation mode for MSE<M_{TL-3}. For the scenario 1 it happens after 18300 symbol intervals.

C. Effective Performance: Equalization Success Index

The evaluation results of ESI are presented in Table 1. The ESI indexes are the result of 1000 independent Monte Carlo tests lasting 50000 symbol intervals. Based on these results it can be seen that the improved Soft-DFE solution (results 2) shows significantly better performance then the original one (results 1). Also, it is confirmed that the RC-PLL presents a much higher robustness in scenario 1. It should be noted that Soft-DFE in scenario 2 (results 3) attains the lowest ESI for the channel MP-B. It is because the channel Mp-B shows a deep asymmetric attenuation in the short range around the carrier frequency of 1800 Hz (see Fig. 2).

TABLE I
ESI in [%]: Mp with SNR=30 dB and offset = (0.0-0.24) Hz

Mp-channel	A	B	C
1. CJEM-W, RC-PLL-ON	99.7	69.6	57.8
2. CJEM-TL, RC-PLL-ON	99.8	98.3	99.3
3. CJEM-TL, RC-PLL-OFF	92.1	77.2	91.3

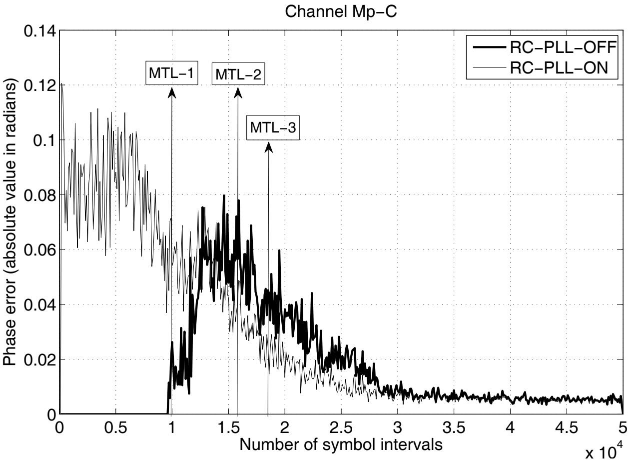

Fig. 4. Phase error convergence of RC-PLL averaged over 20 independent runs, SNR=30dB, offset = (0.0-0.24) Hz.

V. CONCLUSION

In this paper, the blind self-adaptive Soft-DFE scheme for 64-QAM systems is proposed. Using computer simulations, it is shown that the two sources of convergence instability, which are detected in the case of high-order QAM signal constellations, can be successfully eliminated by introducing both the tap-leakage entropy-based decorrelator and the classical decision-directed PLL based on the reduced 64-QAM constellation. In this way, the structure-criterion switching ability of the original Soft-DFE scheme is preserved with a practically neglected increase in complexity.

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