

A New Decorrelation Algorithm for a Soft Decision Feedback Blind Equalizer

Vladimir R. Krstic¹, Zoran Petrovic²

Abstract – This paper proposes a new decorrelation algorithm for a soft decision feedback blind equalizer (Soft DFE) based on a DFE switching structure model. The algorithm is developed using joint entropy maximization (JEM) framework suitable for introducing a desirable nonlinearity directly into the stochastic gradient algorithm. Simulation results have shown that an efficient decorrelation based on JEM algorithm provides a better convergence rate and switching robustness than the existing hard DFE solution.

Keywords – Joint entropy maximization decorrelator, soft decision feedback blind equalizer

I. INTRODUCTION

The decision feedback equalization (DFE) is a technique widely used for removing intersymbol interference in channels with severe amplitude distortions. The well known conventional solutions of DFE, based on supervised training and stochastic gradient algorithms, attain a good cost-performance ratio for many different systems with slowly varying channels. On the other hand, in systems where such training may not be possible or desirable, e.g., some broadband access systems [1], the corresponding blind tap update procedures have to be used. Unfortunately, the existing blind algorithms, originally designed for transversal equalizers [2], [3], cannot be directly applied with a recursive equalizer, such as a DFE, because of the phenomenon of error propagation that characterizes a decision feedback updating. Namely, the enormous number of errors at the start of equalization restricts the use of blind adaptation to the case of an initially well “open eye” corresponding to a mild channel. Recently, several authors have presented various approaches to overcome this major defect of decision feedback blind equalizers [4], [5], [6].

This paper presents a new decorrelation algorithm for the soft decision feedback blind equalizer (Soft DFE), which is designed as a combination of the soft feedbacks based on joint entropy maximization (JEM) criteria [5], and the DFE solution switching from the cascade of decoupled linear components (**G-R-T**) to a conventional DFE (Hard DFE) [4]

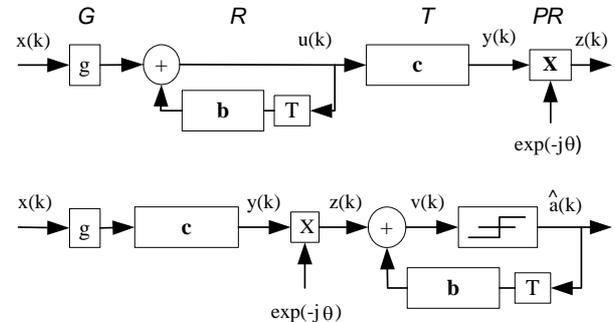


Fig. 1. Structure of Soft DFE: decomposed DFE consists of **G**-gain control, **R**-decorrelator, **T**-transversal FS equalizer and **PR**-phase rotator (above) and Hard DFE (below)

in Fig. 1. The key feature of Soft DFE is a new JEM algorithm for decorrelator (whitening filter) introduced into the existing cascade to provide a better conditioning of the transversal equalizer, which utilizes Godard’s [2] or constant modulus algorithm (CMA) than in the case when the algorithm suggested in [4] is applied.

The proposed Soft DFE is a pass-band T/2 spaced equalizer, which in the steady state works as a conventional minimum mean square error (MMSE) decision directed DFE. On the other hand, at the beginning the Soft DFE starts as a linear zero forcing (ZF) equalizer composed of a kind of automatic gain control **G**, decorrelator **R** and transversal CMA equalizer **T** in the given order. The new decorrelator **R** is implemented by two decoupled all-pole recursive whitening filters (WF) working in parallel in such a way that one of them processes even samples, while the other processes odd samples, once per a symbol interval T. When the MSE of **T** attains the defined threshold level MSE_{TL} the cascade switches into the decision feedback structure so that the coefficients of any of two WF become the coefficients of the feedback part of Soft DFE. The tap update procedure of this feedback is also based on a JEM-type algorithm combining the hard decision with the soft decision [5]. Practically, Soft DFE has two complementary JEM feedbacks: the first that works as the decorrelator during the blind acquisition, and the second that is active after the structure switching.

The primary goal of this paper is to examine the effects of smoothing parameter of JEM decorrelator on CMA equalizer and to evaluate the performance of Soft DFE with respect to existing Hard DFE. Section II generates a new JEM algorithm for a purely recursive whitening filter. Section III addresses the selection problem of the smoothing parameter of JEM algorithm. Section IV presents the simulation results of comparative performance testing of both Hard and Soft DFE solution.

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II. JEM ALGORITHMS

The basic gradient recursion for the soft decision feedback model in Fig. 2a. has been derived in [5] for BPSK system and is given by

$$b_i(k+1) = b_i(k) - \mu r(k)r(k-i) \quad (1)$$

where $b_i(kT) = b_i(k)$, $i = 1 \dots N$, are feedback coefficients, and μ is a positive step size. The stochastic gradient in (1) is a function of nonlinearity applied in the soft decision device, $g(v(k)) = \alpha \cdot \tanh(\beta v(k))$, where $v(k)$ and $r(k)$ are real data input-output sequences of the soft decision device, and α and β are positive constants.

The direct implementation of above stochastic gradient algorithm can be very complex for two-dimensional systems therefore it is practical to derive the specific acceptable realizations using nonlinearity approximations. One of the possible variants of Eq. (1) can be derived if we use Taylor series approximation for $r(k)$ and $r(k-i)$ in (1), in the following way:

$$r(k) = \alpha \cdot \tanh[\beta \cdot v(k)] = \alpha\beta v(k) - \frac{\alpha\beta}{3} v(k)^3 + \dots \quad (2)$$

$$r(k)r(k-i) \approx \left[\alpha\beta v(k) - \frac{\alpha\beta}{3} v(k)^3 \right] \cdot \alpha\beta v(k-i) \quad (3)$$

$$b_i(k+1) = b_i(k) - \mu_w v(k) \cdot \left[1 - \beta_1 v(k)^2 \right] \cdot v(k-i) \quad (4)$$

where $\mu_w = \mu\alpha^2\beta^2$ is a step size, and $\beta_1 = \beta^2/3$ is a positive constant. In the complex version of Eq. (4) the variable $v(k)^2$ is replaced with $|v(k)|^2$.

Using the above approximation the basic JEM recursion (1) is transformed into a new JEM-type decorrelation algorithm (JEM-W) for the whitening filter shown in Fig. 2b where $v(k) = u(k)$. It should be noted that the parameter β_1 in Eq. (4) can be used to smooth the error $e(k) = u(k)[1 - \beta_1 u(k)^2]$, i.e., to vary the algorithm characteristics. This quality of β_1 will be discussed in the next section.

Because the \mathbf{R} is a recursive filter its stability has to be the subject of additional investigation. However, for the purpose of this work we can stress that the results of extensive testing of JEM-W by a software simulator have exhibited strong robustness concerning stability. This indicates that JEM-W has preserved the so-called "self-stabilization" property of the existing decorrelation algorithm that has been applied in [4] where $e(k) = u(k)$ is a prediction error.

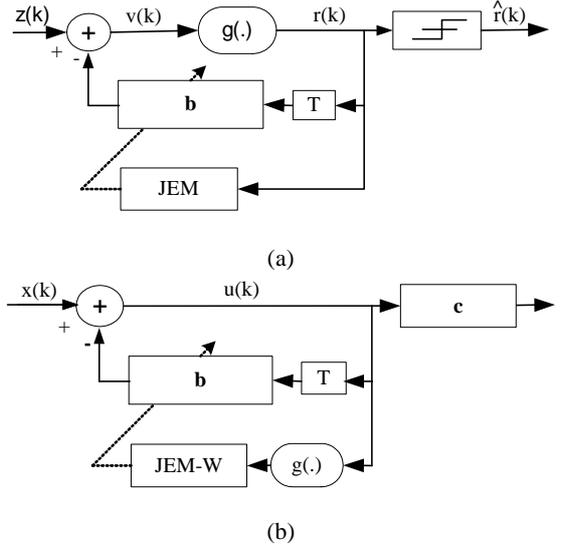


Fig. 2. (a) Basic JEM DFE model, (b) JEM decorrelator

III. SMOOTHING PARAMETER SELECTION

We have based the smoothing parameter selection on the kurtosis method, i.e., Shalvi-Weinstein theorem [3] and the decomposed DFE model elaborated in [4]. Thus, Soft DFE as well as its counterpart Hard DFE, represents ZF equalizer during the blind mode, which competes to maximize the normalized kurtosis of output sequences $K(z(k))$, or equivalently to maximize the function $F(\mathbf{s})$

$$\frac{K(z(k))}{K(a(k))} = F(\mathbf{s}) = \left[\frac{\|\mathbf{s}\|_4}{\|\mathbf{s}\|_2} \right]^4, \quad \|\mathbf{s}\|_p = \left[\sum_i |s_i|^p \right]^{1/p} \quad (5)$$

where $a(k)$ and $z(k)$ are the system input-output complex data sequences, and \mathbf{s} denotes the vector corresponding to the impulse response of a noiseless system, (channel + equalizer). However, because a channel is unknown, we can recall a one-to-one correspondence between stationary points in the \mathbf{s} and \mathbf{c} domains [7] that suggests to evaluate $F(\mathbf{s})$ in Eq. (5) with respect to vector \mathbf{c} calculating only the equalizer vector length $\|\mathbf{c}\|$. According to that we have introduced the following relations for the normalized kurtosis ratio of \mathbf{R} and \mathbf{T}

$$K_R(u_s) = \left[\frac{\|\mathbf{b}\|_4}{\|\mathbf{b}\|_2} \right]^4 \quad (6)$$

$$K_T(z_s) = \left[\frac{\|\mathbf{c}\|_4}{\|\mathbf{c}\|_2} \right]^4 \quad (7)$$

where K_R and K_T are the kurtosis ratios of the outputs of \mathbf{R} and \mathbf{T} , respectively, at the moment of reaching switching threshold MSE_{TL} , $u_s = u(k_s)$ and $z_s = z(k_s)$, whereas $\|\mathbf{b}\|$ and $\|\mathbf{c}\|$ are the lengths of vector coefficients of \mathbf{R} and \mathbf{T} .

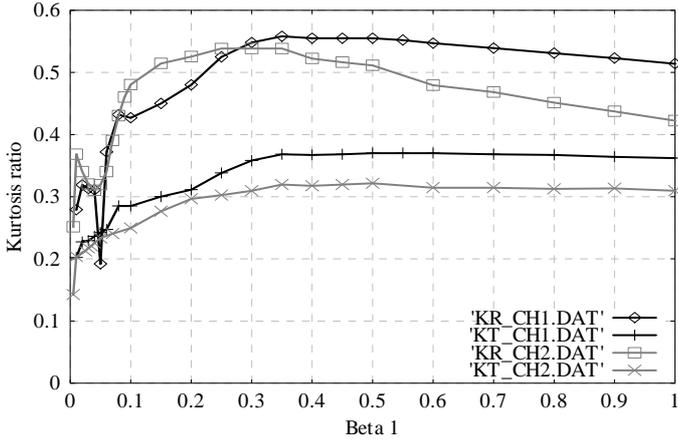


Fig. 3. Kurtosis ratio for Soft DFE: Channel 1 $K_R = KR_CHA1$, $K_T = KT_CH1$ and Channel 2 $K_R = KR_CHA2$, $K_T = KT_CH2$

The described kurtosis method is used to evaluate the influence of parameter β_1 to startup characteristics of equalizer T . The corresponding analysis is performed by software simulator of the ITU-V.32 modem running $N_{\text{RUN}} = 100$ Monte Carlo tests. The curves in Fig. 3 present both K_R and K_T kurtosis as a function of β_1 , obtained by 32QAM signal constellation and two different channels with severe linear distortions and white additive noise SNR=30dB. These results show that JEM-W decorrelator has substantially influenced the equalizer performance. Also, it indicates that it is possible to select the range of compromise values β_1 where we can reasonably expect the best equalizer performance for different channels in the observed system. Evidently, for smaller values of β_1 (e.g., $\beta_1 < 0.4$) the desired effects of the JEM-W decorrelator begin to diminish. On the other hand, in the range where the kurtosis K_T reaches maximum values we can attain the desired improvements of equalizer convergence characteristics. According to that we have roughly estimated the range ($0.4 < \beta_1 < 0.6$) as a range of our interest. Finally, for ($0.6 < \beta_1 < 1.0$) we have verified the degradation of MMSE characteristics of Soft DFE.

IV. PERFORMANCE TESTING

In this section we present the testing results of Soft/Hard DFE solutions, carried out by a software simulator for two channels. The transfer function of applied transmitter and receiver filters follows a raised cosine with a roll-off factor 0.12. The voice-band non-minimum phase channels are implemented combining their transfer functions with the transfer function of the transmitter filter. Channel 1 represents a three-ray multipath environment [5], whose impulse response is given by

$$h(t) = p(t) \cdot W(t) + 0.8p(t-0.25T) \cdot W(t-0.25T) + 0.4p(t-2T) \cdot W(t-2T) \quad (8)$$

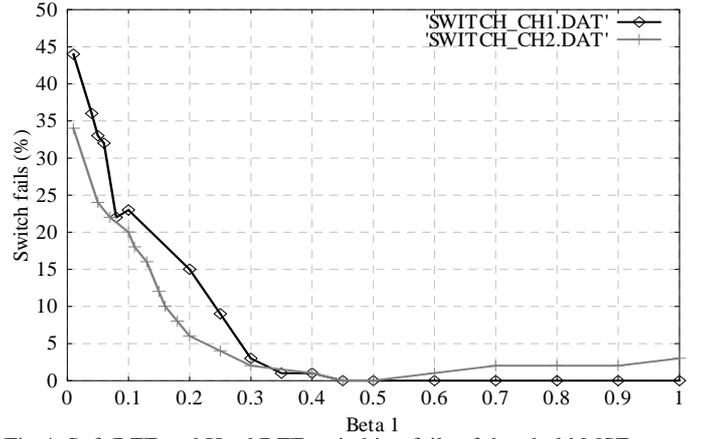


Fig.4. Soft DFE and Hard DFE switching fails of threshold $MSE_{TL-1} = 1.14\text{dB}$, SNR=30dB: Switch1_ch1 for Channel 1and Switch1_ch2 for Channel 2

where $p(t)$ is the basic pulse shape and $W(t)$ is a rectangular window spanning $[-16T, 16T]$. Channel 2 is well known 3002 channel [2]. The CMA equalizer has 42 coefficients and each whitening filter has 6 coefficients. Soft/Hard DFE equalizers have the centered reference tap, which takes initial value 3.1 for both channels. The switching threshold is set up to $MSE_{TL} = 1.14$ dB that corresponds to totally “closed” signal constellation.

Test 1 presents the performance of structure switching of Soft/Hard DFE for 32QAM signal constellation. The test has been performed by counting the unsuccessful switching from a blind to decision directed DFE mode during N_{RUN} tests, where each run was limited to $3 \times 10^4 T$ intervals. In the case of Soft DFE the counting is carried out for different values of parameter β_1 and these results are shown in Fig. 4. Evidently, Soft DFE switching performance (threshold reaching) is correlated with the results showing kurtosis versus β_1 . The results of Test 1 have confirmed that, for selected range, the Soft DFE exhibits a much better switching performance than Hard DFE solution, which fails in 18 % for Channel 1 and 17 % for Channel 2.

Test 2 presents the Soft/Hard DFE MSE performance for both 16-QAM and 32-QAM system. The N_{RUN} tests have been carried out for $\beta_1 = 0.50$ that was selected as a compromise value for many different channels represented by Channel 1 and Channel 2. Fig. 5 presents that Soft DFE shows much better convergence characteristics in comparison to Hard DFE: 4.7 dB, 7.8 dB, 3.0 dB and 5.6 dB for 16QAM_CH1, 32QAM_CH1, 16QAM_CH2 and 32QAM_CH2, respectively. Another gain of Soft DFE in comparison to Hard DFE concerns performance convergence of both 16-QAM and 32-QAM system.

V. CONCLUSION

Soft DFE is improved variant of the existing DFE that a blind acquisition completes as a linear ZF equalizer and then switches into a decision directed model at the moment the risk of error propagation is dramatically mitigated. That is dominantly gained by JEM decorrelator introduced into the existing cascade, which speedup blind acquisition and provides a better initial coefficients setup for decision directed operation mode. The presented simulation results for 16QAM and 32QAM signal constellations have shown we can reach significantly better convergence rate and MMSE in respect to existing DFE.

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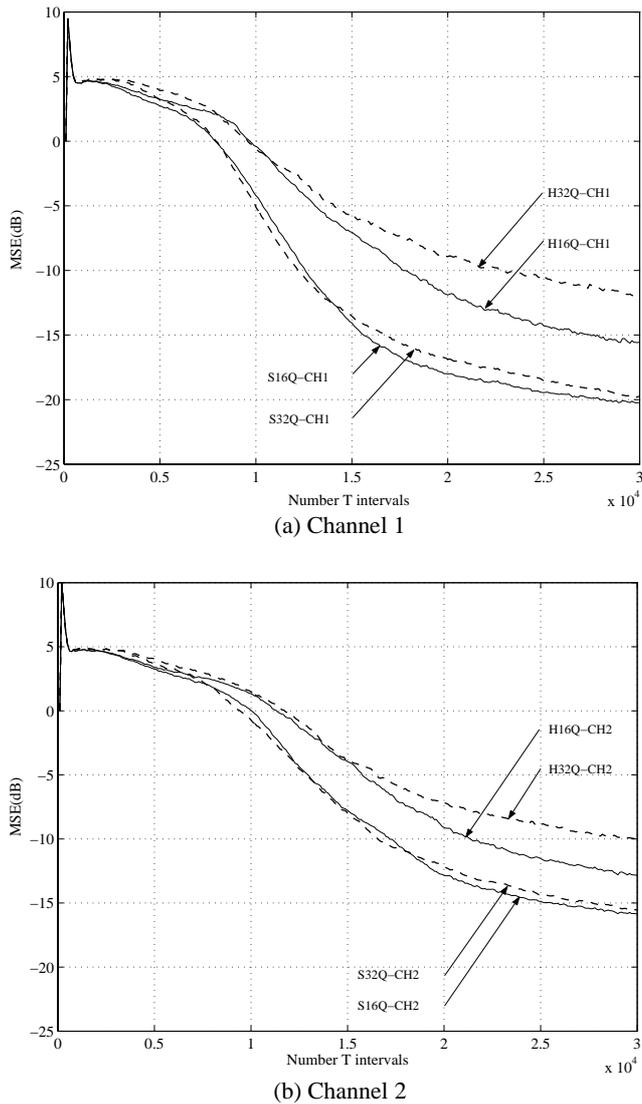


Fig. 5. MSE comparison of two solutions of blind equalizers, Hard DFE and Soft DFE for 16-QAM and 32-QAM with Channel 1 and Channel 2