On Structure-Criterion Switching Control for Self-Optimized Decision Feedback Equalizer

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***Abstract*—This paper proposes a new structure-criterion switching control method for the self-optimized blind decision feedback equalizer (DFE) which switches operation modes according to the mean square error (MSE) convergence state. The new operation switching control shortens the blind operation period time of the DFE and, hence, speeds up its effective convergence rate. This is achieved combining the MSE estimate of the DFE's output and a posteriori error of the DFE's recursive filter acting as the front-end all-pole amplitude equalizer during the blind operation mode. The efficiency of the improved DFE switching control is verified by the software simulator using QAM signals and multipath channels.**

***Index Terms*—Blind equalization, decision feedback equalizer, Joint Entropy Maximization criterion, variable threshold.**

I. Introduction

BLIND equalizers commonly complete their convergence through two operation modes. They begin operation with a blind acquisition of the received signal and then, depending on the convergence state or signal quality, switch adaptation to the decision-directed operation mode that should guarantee the completeness of their convergence process. In such scenario, searching for optimal switching conditions, an equalizer estimates in line some measure of convergence quality and employs a suitable performance threshold level to decide the optimal switching time [1], [2].

Let us consider a blind equalizer estimating in line the output mean square error (MSE) which characterizes its convergence process by the following MSE states: 1) $MSE_{B,opt}$ is the mean square error achievable during the blind acquisition period if the equalizer's coefficients reached the optimal setup, 2) $MMSE_{opt}$ is the mean square error achievable during the decision-directed (DD) operation mode if the equalizer's coefficients reached the optimal steady-state setup and 3) $\mathrm{MSE}_{\mathrm{TLF}}$ is a fixed user-selectable threshold level deciding the equalizer switching from the blind to the DD operation mode. Accordingly, the three typical equalization scenarios are possible. If we assume the equalizer can reach the optimal coefficients setup in the blind mode then: 1) for $MSE_{B,opt} = \mathrm{MSE}_{\mathrm{TLF}}$ the equalizer will successfully switch operation to the DD operation mode, 2) for $\mathrm{MSE}_{\mathrm{TLF}} < MSE_{B,opt}$ the equalizer will not switch itself to the DD mode and the equalization failure is result and 3) for $\mathrm{MSE}_{\mathrm{TLF}} > MSE_{B,opt}$ the equalizer will switch operation to the DD mode faster than in the case 1) but it certainly will not reach the optimality $MMSE_{opt}$ and even some pathological states are possible. Based on these scenarios it is obvious that the successfulness of equalizer operation mode switching control depends on the selected threshold $\mathrm{MSE}_{\mathrm{TLF}}$ which is, generally, unknown and depends on the number of system and equalizer parameters. In order to facilitate the threshold level selection issue, in this paper we have focused our attention on the last scenario which motivates the question if we can select higher threshold levels $\mathrm{MSE}_{\mathrm{TLF}}$ than $MSE_{B,opt}$ aiming at faster equalizer convergence rates.

To find the answer to the above question we have addressed the blind Soft-DFE equalizer of QAM (quadrature amplitude modulated) signals [3] and utilized the whitening capability of its Joint Entropy Maximization (JEM) decorrelator [4] to create a new structure-criterion switching control which speeds up the equalizer's convergence rate.

II. JEM whitening algorithm

In the following text, after a short description of the Soft-DFE three steps operation, the JEM whitening algorithm is recalled [4] in order to present a posteriori error method which has the double function of adapting the whitener coefficients leaky factor and compensating for an insufficiency of the MSE estimates during the blind mode. The interested reader can find more details of the Soft-DFE criteria and algorithms in references [3] and [4].

The Soft-DFE converges through three operation modes - blind, soft-transition and tracking – to reach a steady state operation, Fig, 1. At the start of the blind mode the Soft-DFE transforms its original DFE structure into the cascade of four linear signal transformers: the gain control (*GC*), all-pole decorrelator (whitener) *WT* of the received signal, blind equalizer (*TE*) and phase rotator (*PR*) including signal demodulation function, Fig. 1a. Effectively, in the blind mode the Soft-DFE acts as a T/2 fractionally spaced CMA equalizer (T/2-FSE, T is a symbol period) [5] which divides blind equalization task between four signal transformers. In the next soft-transition mode the equalizer continues operation as the DD-DFE using LMS and JEM algorithms, respectively, in its linear and nonlinear parts and, finally, in the tracking mode

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Proceedings of 4th International Conference on Electrical, Electronics and Computing Engineering, IcETRAN 2017, Kladovo, Serbia, June 05-08, ISBN 978-86-7466-692-0 pp. TEI1.5.1-5

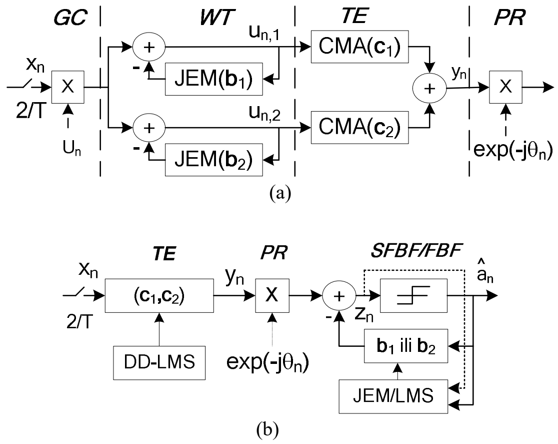


Fig. 1. Soft-DFE structure-criterion transformation: (a) blind mode and (b) soft-transition mode (*SFBF* with JEM, dotted line) and tracking mode (*FBF* with DD-LMS, presented only by solid lines).

the Soft-DFE transforms itself back into the original DFE entirely controlled by the DD-LMS algorithm, Fig. 1b.

The JEM whitening algorithm with the variable coefficient leaky regularization (JEM-VL) [4] is given by

$$u_{i,n} = x_{i,n} - \mathbf{b}_{i,n}^T \mathbf{u}_{i,n}\,,\ i = 1,2 \tag{1}$$

$$\mathbf{b}_{i,n+1} = \mathbf{b}_{i,n} - \gamma_n \mathbf{b}_{i,n} - \mu_W u_{i,n}(1-\beta_W \left|u_n\right|^2)\mathbf{u}_{i,n}^* \tag{2}$$

where $\mathbf{u}_{i,n} = [u_{i,n,1},...,u_{i,n,N}]^T$ and $\mathbf{b}_{i,n} = [b_{i,n,1},...,b_{i,n,N}]^T$ are, respectively, whitener's regression and coefficient vectors, $\gamma_n \geq 0$ is the time-variable leaky factor, β_W is the free parameter representing the slope of the employed neuron function [3], μ_W is a small positive step-size and N is the span of the whitener delay line in T periods. The specific of the JEM-VL algorithm, besides the slope β_W controlling its entropic capability, is its variable leaky factor γ_n. Acting in opposition to the entropy-gradient, the leaky term $\gamma_n b_n$ decreases the magnitude of whitener coefficients avoiding superfluous coefficients to disturb the equalizer convergence process at the time of its switching from the blind to decision-directed operation mode. Using the variable leaky factor instead of the fixed one, the trade-off is achieved between its ability to prevent a coefficients overgrowth and its capacity to force a biased coefficients setup.

The adaptation of the leaky γ_n is based on the analysis of whitener's a posteriori errors and the heuristic punish/award rule [6] deciding when and how much to increase or decrease the leaky factor. Accordingly, the leaky adaptation rule in JEM-VL comprises the following three steps: the calculation of a posteriori error with ($\gamma > 0$) and without ($\gamma = 0$) coefficient leakage, decisions when and decisions how much to increase or decrease leaky. The a posteriori error $\tilde{e}_n^{VL}$ estimate for $\gamma > 0$ in JEM-VL is given by

$$\tilde{u}_n = x_n - \mathbf{b}_{n+1}^T \mathbf{u}_n \tag{3}$$

$$\tilde{e}_n^{VL} = \tilde{u}_n (1-\beta_W \left|\tilde{u}_n\right|^2) \tag{4}$$

and the corresponding a posteriori error $\tilde{e}_n^W$ estimate for $\gamma_n = 0$ in (2) (corresponds to the original whitening algorithm JEM-W) is given by

$$\mathbf{b}_{n+1} = \mathbf{b}_n - \mu_W u_n (1-\beta_W \left|u_n\right|^2)\mathbf{u}^* \tag{5}$$

$$\tilde{u}_n = x_n - \mathbf{b}_{n+1}^T \mathbf{u}_n \tag{6}$$

$$\tilde{e}_n^W = \tilde{u}_n (1-\beta_W \left|\tilde{u}_n\right|^2)\,. \tag{7}$$

It should be noted that the both a posteriori errors, $\tilde{e}_n^{VL}$ in (4) and $\tilde{e}_n^W$ (7), are obtained using the same current value of the whitener input x_n; in above relations the index i is dropped for the purpose of simplicity.

In the next step, based on the comparison of achieved a posteriori errors, the "if-else" relation

if $\tilde{e}_n^{VL} > \tilde{e}_n^W$ then
  set $m_{n+1} = \max(m_n - l_d, 0)$
else
  set $m_{n+1} = \min(m_n + l_u, M)$
end if (8)

decides when to decrease or increase the leaky factor and, finally, the quantized function

$$\gamma_n = f(m_n) = \gamma_{\max}(m_n / M) \tag{9}$$

calculates how much to decrease or increase the leaky factor; in relations (8) and (9) $m_n = 0,...,M$ is an independent variable and $(M, l_d, l_u) \in \mathbb{Z}$, $\gamma_{\max} \in \mathbb{R}$ and m_0 are user-definable parameters.

III. Switching Control with Variable MSE Threshold

As emphasized in introduction, in order to decide the best switching moment from the blind to the DD soft-transition mode, the Soft-DFE in line estimates the MSE of symbol estimates y_n and compares it with a selected threshold MSE_{TLF} level. Using the constant modulus error

$$e_C(n) = \left|y_n\right|^2 - R_C \tag{10}$$

defined for the CMA criterion [7], the Soft-DFE estimates the MSE at the output *TE* using the recursion

$$MSE_{B,n} = \lambda \cdot MSE_{B,n-1} + (1-\lambda)\left(\left|y_n\right| - \sqrt{R_C}\right)^2 \tag{11}$$

where R_C is the constant representing the fourth-order statistics of the applied QAM signal and λ is the forgetting factor which determines the MSE estimation quality; λ is less than one and typically $\lambda = 0.99$. During the DD operation modes (soft-transition and tracking), the Soft-DFE exploits the same MSE estimation principle in (11) but employs the error $|z_n - \hat{a}_n|$ instead of $\left(|y_n| - \sqrt{R_C}\right)$.

The Soft-DFE switching method based on the estimate $MSE_{B,n}$ suffers from several weaknesses; let us analyze them in more details. First, having in mind that we don't know the optimal $MSE_{B,opt}$, the selection of the threshold $\mathrm{MSE_{TLF}}$ is a meter of some heuristic. Second, the $MSE_{B,n}$ given by (11) is a crude estimate of the MSE for all non-constant modulus QAM signals. The quantity $\left(|y_n| - \sqrt{R_C}\right)$ is not the error but rather the dispersion of the modulus of symbol estimates with respect to the constant $\sqrt{R_C}$. Besides, the $MSE_{B,n}$ aggregates the convergence state of the cascaded Soft-DFE (Fig. 1a) with the dominate influence of the *TE*, i.e., CMA algorithm relying on the R_C. Thus, the $MSE_{B,n}$ doesn't reflect directly the influence of the second-order statistic of the given signal being recovered by the whitener *WT*.

In order to compensate for $MSE_{B,n}$ insufficiencies, we have combined the existing fixed threshold $\mathrm{MSE_{TL}}$ with the whitener's a posteriori error $\tilde{e}^{VL}_{i,n+1}$ and introduced it into the Soft-DFE switching control. As a result the variable threshold is obtained [8] given by

$$\mathrm{MSE_{TLV}} = \mathrm{MSE_{TL}} - S(e^{VL}_{1,n+1} + e^{VL}_{2,n+1}) \qquad (12)$$

where S is a small positive scaling factor and $\mathrm{MSE_{TL}}$ is a fixed term. Practically, by using the whitener's a posteriori error as a variable threshold term we have created the new equalizer switching control that reflect directly the recovering of both second-order and four-order statistics of the applied signal. Using the variable threshold, the equalizer structure-criterion switching control responds as follows: for a lower a posteriori error the $\mathrm{MSE_{TLV}}$ becomes higher, which shortens the blind equalization time and, hence, speeds up the equalizer convergence rate, and reverse, for a higher a posteriori error the $\mathrm{MSE_{TLV}}$ becomes lower which lengthens the blind acquisition time and slows the equalizer convergence. Effectively, in such a way more accurate MSE estimation is achieved which allows to apply in (12) higher fixed terms $\mathrm{MSE_{TL}}$ than $\mathrm{MSE_{TLF}}$.

To avoid the false equalizer switching through the operation modes, which could be caused by the non-stationarity of the MSE data, the Soft-DFE switching control implementation is based on the multiple checking of the threshold level passage. According to the switching rule

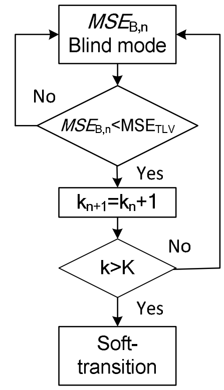


Fig. 2. Soft-DFE rule switching from blind to soft-transition mode.

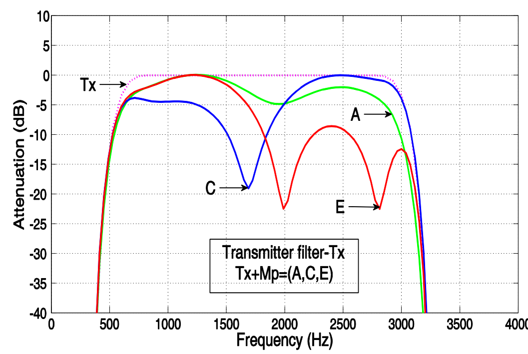


Fig. 3. Normalized attenuation characteristics of Mp-(A, C, E) channels.

presented in Fig.2, the equalizer is allowed to switch from the blind to the soft-transition mode if and only if the $MSE_{B,n}$ satisfies $MSE_{B,n} < \mathrm{MSE_{TLV}}$ during the K equalizer's update iterations where K is user-definable integer larger than 1. The same switching rule is valid for the Soft-DFE switching from the soft-transition to the tracking operation mode.

IV. Simulation Results

The efficiency of the new structure-criterion switching control is verified by comparing the Soft-DFE performance achieved with the variable MSE_{TLV} (TLV) and fixed MSE_{TLF} (TLF) thresholds. The simulation tests are carried out using the single-carrier QAM system transmitting the 16-,[64]-QAM signal over multi-path channels Mp-(A, C, E) with the 25, [30] dB signal-to-noise ratio; the amplitude characteristics of channels are presented in Fig. 3. The Soft-DFE's filter dimensions and user-definable parameters are selected as follows. The delay line spans of *WT* and *TE* are, respectively, 5 T and 23, [24] T and the initial values of their coefficients are all zero except of the *TE* referent (double-spike) coefficients $c_{1,r} = c_{2,r} = 1.0$. The maximum leaky factor $\gamma_{\max}$ for 16-,[64]-QAM is 2^{-12} [2^{-11}], and other leaky parameters $\{ l_d = 5, l_u = 40,\ m_0 = 40,\ M = 400 \}$ are the same for both 16- and 64-QAM signals. The algorithm step-sizes are changed through three operation modes as follows: $\mu_{TE-1} = 2^{-16}[2^{-21}]$, $\mu_{TE-2} = 2^{-15}[2^{-20}]$, $\mu_{TE-3} = 2^{-13}[2^{-16}]$ $\mu_{W} = 2^{-19}[2^{-22}]$, $\mu_{SFBF-JEM} = 2^{-18}[2^{-21}]$, $\mu_{FBF-LMS} = 2^{-14}[2^{-13}]$. The leaky parameters and step-sizes are chosen in a way to achieve the best compromise between the convergence rate and the equalization successfulness.

The results of tests are given in the terms of the probability density histograms of blind acquisition period time in T intervals, the equalization successfulness index (ESI) given as the ratio between the successful equalizations and the total number of Monte Carlo runs and the MSE convergence characteristics; the presented histograms and ESI indices are obtained for 10000 and MSE convergence curves for 200 independent Monte Carlo runs. The switching control parameters and the neuron slope β_W, which are utilized for final fitting the switching control efficiency are given as follows:

16-TLF={ $\text{MSE}_{\text{TLF},16} = 1.355$, $\beta_{W,16} = 7$, K=95},

16-TLV={ $\text{MSE}_{\text{TL},16} = 1.7$, $\beta_{W,16} = 9$, S=0.00145, K=105},

64-TLF={ $\text{MSE}_{\text{TLF},64} = 6.340$, $\beta_{W,64} = 2.4$, K=95},

64-TLV={ $\text{MSE}_{\text{TL},64} = 7.26$, $\beta_{W,16} = 2.8$, S=0.00165, K=105}.

The histograms in Figures 4 and 5 for both 16- and 64-QAM signals demonstrates smaller Mean and STD (standard deviation) values of the blind acquisition time for TLV than for TLF case; the accurate values of Mean and STD are given in TABLE I. The influence of the TLV switching control on the effective equalizer convergence rate is presented in Figures 6 and 7 where can be seen that using of variable threshold provides higher convergence rates independently of given signal and channels characteristics. For the purpose of comparison correctness, the control switching parameters {MSE_{TL},S,K} in the TLV case are selected in such a way to reach approximately the same ESI indices as in the case of TLF, TABLE II. Besides, it should be noted that the variable threshold method allows the use of higher values of both the fixed term MSE_{TL} and the slope β_W than for the fixed threshold case without sacrificing ESI and residual MSE equalizer performances.

TABLE I
BLIND ACQUISITION PERIOD TIME STATISTICS [T] FOR 16-, 64-QAM

Channel	Mp-A	Mp-C	Mp-E
	16-QAM		
Mean: TLF	4179	5487	4441
STD: TLF	984	1456	951
Mean: TLV	3773	4193	3605
STD: TLV	573	633	324
	64-QAM		
Mean: TLF	8495	10929	9441
STD: TLF	2229	2731	1934
Mean: TLV	8161	8854	7804
STD: TLV	1469	1286	920

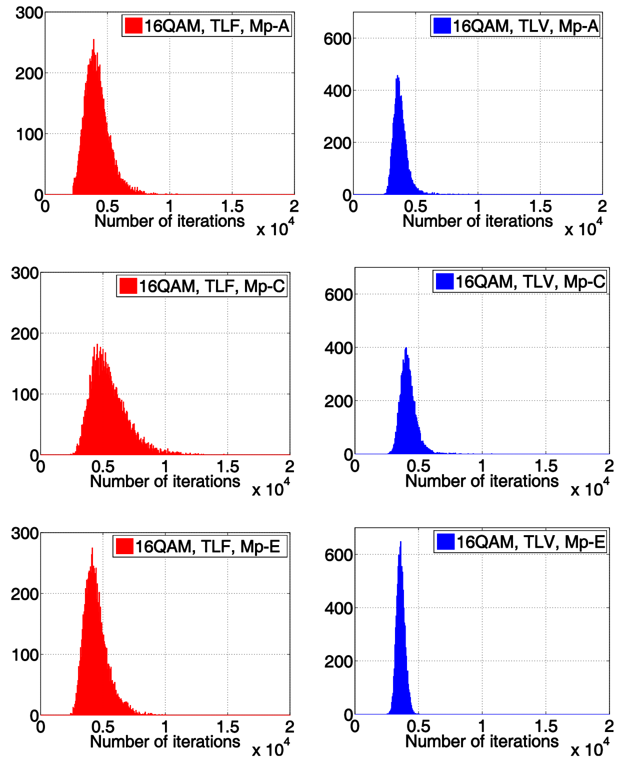


Fig. 4. Blind acquisition histograms obtained for 16-QAM signal and Mp-(A,C,E) channels using fixed (TLF) and variable (TLV) thresholds.

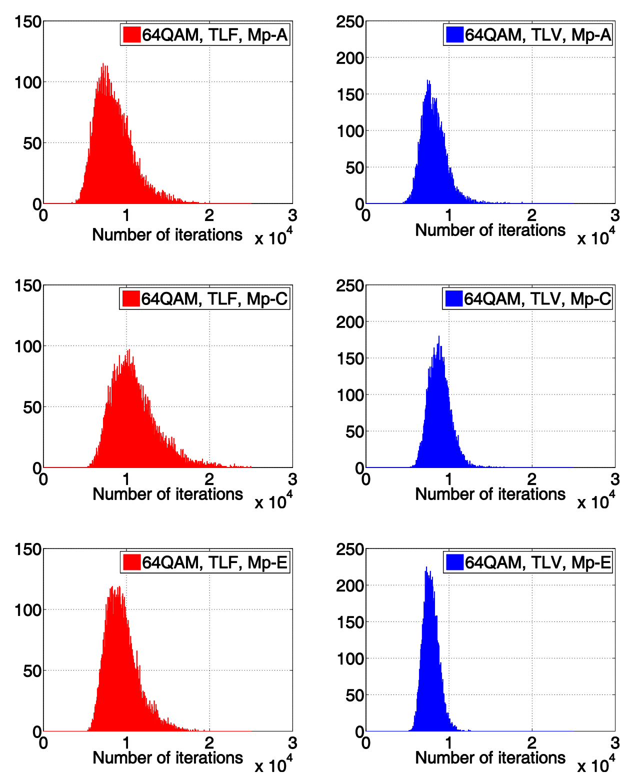
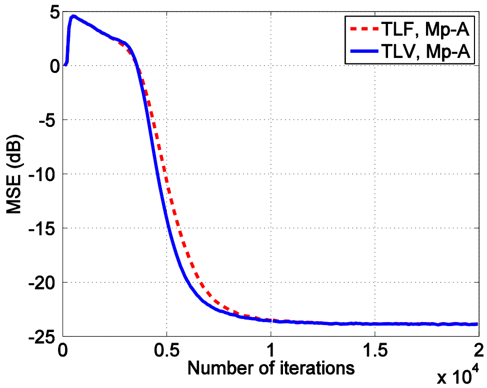


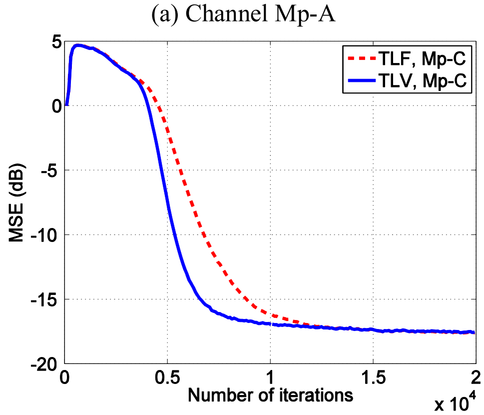
Fig. 5. Blind acquisition histograms obtained for 64-QAM signal and Mp-(A,C,E) channels using fixed (TLF) and variable (TLV) thresholds.

TABLE II
EQUALIZATION SUCCESS INDEX [%] FOR 16-, 64-QAM

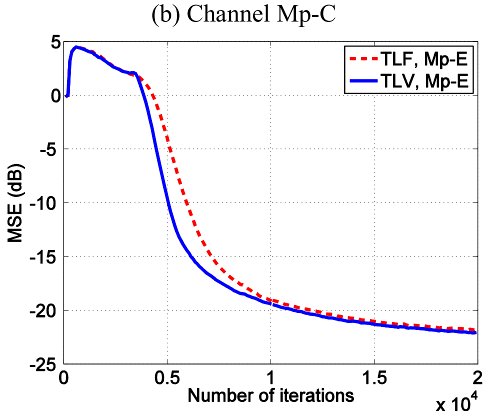
Channel	Mp-A	Mp-C	Mp-E
16-QAM			
ESI: TLF	99.92	99.87	98.94
ESI: TLV	99.94	99.90	99.20
64-QAM			
ESI: TLF	100	99.50	98.40
ESI: TLV	100	99.66	98.10



(a) Channel Mp-A



(b) Channel Mp-C



(c) Channel Mp-E

Fig. 6. Comparison of MSE convergence curves obtained for 16-QAM and Mp-(A,C,E) channels using fixed (TLF) and variable thresholds (TLV).

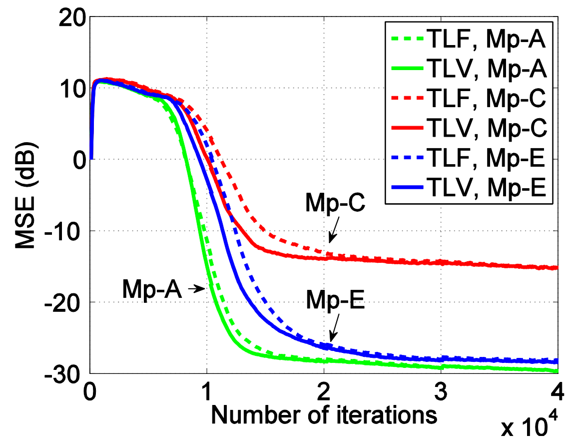


Fig. 7. Comparison of MSE convergence curves obtained for 64-QAM and Mp-(A,C,E) channels using fixed (TLF) and variable thresholds (TLV).

V. CONCLUSION

The structure-criterion switching control based on the variable switching threshold level shortens the equalizer blind acquisition time and, hence, speeds up its effective convergence rate without sacrificing its residual MSE and equalization successfulness performance. Besides, the variable switching control is less sensitive on the threshold parameters selection then the fixed one. The method verified in the case of Soft-DFE can also be applied to other types of blind equalization schemes using a front-end all-pole whitener or similar pre-processing of the received signal.

ACKNOWLEDGMENT

This work was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia; the project of technological development TR 32037, 2011-2016.

REFERENCES

[1] J. Labat, O. Macchi and C. Laot, “Adaptive decision feedback equalization: can you skip the training period?,” *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 921-930, 1998.

[2] G. Ananthaswamy and D. L. Goeckel, “A Fast-Acquiring Blind Predictive DFE,” *IEEE Trans. Commun*., vol. 50, no. 10, pp.1557-1560, 2002.

[3] V. R. Krstić. and M. L. Dukić, “Blind DFE With Maximum-Entropy Feedback,” *IEEE Signal Processing Letters*, vol. 16, no 1, pp. 26-29, 2009.

[4] V. R. Krstić*,* A. M. Stevanović and B. Lj. Odadžić, “A Variable Leaky Entropy-Based Whitening Algorithm for Blind Decision Feedback Equalization”, *Wireless Personal Communications*, 2016, vol. 90, no. 4, October (II), 2016, Springer, DOI: 10.1007/s11277-016-3806-7

[5] Li Y. and Z. Ding, "Global Convergence of Fractionally Spaced Godard (CMA) Adaptive Equalizers," *IEEE Trans. Signal Processing*, vol. 44, pp.818-826, Apr., 1996.

[6] M. Kamenetsky and B. Widrow, “A Variable Leaky LMS Adaptive Algorithm”, in *Proc. Thirty-Eighth Asilomar Conference on Signal, Systems and Computers*, vol.1, pp. 125-126, 2004.

[7] Godard, D. N.: ‘Self-Recovering Equalization and Carrier Tracking in Two-Dimensional Data Communication Systems’, *IEEE Trans. Commun.,* 1980, vol. 18, no. 11, pp. 1867-1875, 1980.

[8] V. R. Krstić, “Fast start-up blind DFE equalizer,” Pending Patent RS, P-2017/0205, Feb. 2017.