Decision Feedback Blind Equalizer with Maximum Entropy

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***Abstract* – In this paper we propose a solution of an unsupervised decision feedback equalizer (DFE) based on switching DFE structure and joint entropy maximization algorithms (JEM). In the blind mode it is the linear Godard's equalizer strongly supported by JEM type decorrelator and in the decision-directed mode the nonlinear equalizer with soft decision feedback.**

***Keywords* – Joint entropy maximization decorrelator, kurtosis method, soft decision feedback blind equalizer**

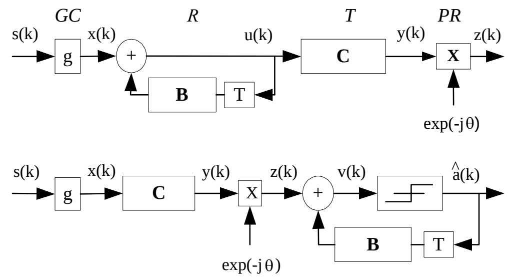

Fig. 1. Blind DFE with "adaptive" structure: linear cascade and classical DFE (below)

I. Introduction

The decision feedback equalization (DFE) is a technique widely used for removing intersymbol interference (ISI) when communication channels introduce severe amplitude distortions. It was common to use the conventional DFE solutions based on minimum mean square error (MMSE) criterion and the transmission of a known data training sequence, which provide a good cost-performance ratio for many different systems. On the other hand, there are many applications where the classical supervised training is not desirable or possible [1]-[4]. Thus, there is a strong interest for unsupervised (blind) equalizers that do not require any pilot symbols. The general problem of blind equalizers is their slow convergence rate to desired filter coefficients setup and the existence of undesirable local minima [5]. Besides, classical DFE using the nonlinear hard decision device (slicer) suffers from the phenomenon of error propagation [6]. For that reason, the blind algorithms (e.g., Godard's [1] or Shalvi-Weinstein [2]), originally developed for linear transversal filters, cannot be directly applied.

Recently, papers [3] and [4] have presented the two different approaches how to skip the supervised training or to overcome error propagation, and, at the same time, to preserve the DFE low computational complexity. The solution [3] (Hard DFE) is based on the "adaptive" DFE structure, which switches between the two modes of operation: the acquisition (blind) mode and the tracking mode. In the acquisition mode, the Hard DFE is decomposed into the cascade consisting of : gain control (***GC***), whitening filter (***R***), linear transversal equalizer (***T***) and phase rotator (***PR***), Fig.1. This cascade constitutes the linear blind equalizer (***LE***) . When the signal constellation at the output of ***T*** is well opened, the ***LE*** switches back into the conventional DFE structure, (Fig.1. below). The basic ideas of this solution are 1) the cascade which splits the difficult task of blind activation into several easier subtasks and 2) observation that the coefficients of recursive part of the all-pole whitening filter are what the conventional DFE needs in its feedback part [6]. On the other hand, in [4] the equalizer with soft decision feedback based on joint entropy maximization (JEM) criterion, has been proposed. The main result of this investigation is a new class of JEM stochastic gradient algorithms whose characteristics depend on a nonlinear function section for decision device.

The DFE blind equalizer presented in this paper is improved and innovated version of the Hard DFE. This new solution (Soft DFE) [7] is developed as the combination of the "adaptive" DFE structure and the JEM type algorithms. Namely, we have retained the basic modes of operation of Hard DFE but the most critical and demanding phases of the blind activation, (the decorrelation of input sequence and transition from acquisition to tracking mode) are optimized by entropy maximization procedures. First, we have applied a new JEM type algorithm for the whitening filter adjustment. Second, the additional mode is introduced (JEM-DFE), which is a short transition between the blind and tracking modes. The main idea underlying the proposed innovations is observation that severe amplitude distortions have to be compensated with the whitening filter optimized by a maximum entropy criterion that will provide the best fit to sharp spectral features of receiving signal [8]. Beside, the error propagation effects can be mitigated using JEM-DFE scheme immediately after structure switching.

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II. SOFT DFE: STRUCTURE AND ALGORITHMS

The Soft DFE is a pass-band fractionally spaced (FS) equalizer implemented by T/2 tap delay lines. It is well known scheme commonly used for joint adaptive equalization and carrier phase tracking in two-dimensional systems. In this paper we have presented only the acquisition and JEM-DFE modes. Soft DFE in the tracking mode is MMSE decision-directed DFE.

A. Acquisition mode

A.1.Automatic gain control ***GC*** is a one-coefficient real equalizer [3], Fig.2.

$$x\left(k-p/2\right)=g(k-1)s\left(k-p/2\right) \qquad (1)$$

$$G(k)=G(k-1)+\mu_G[\sigma_a^2-\left|u(k)\right|^2],\quad g(k)=\sqrt{\left|G(k)\right|} \qquad (2)$$

where $s(k-p/2)\triangleq s(kT-pT/2)$, p=0,1, is the input, g(k) is control signal and the μ_G is a small positive step size. The signal g(k) is estimated once per T. For instance, in the case of transmission signals 16-QAM and 32-QAM taking equiprobable values $\{a(k)\}$ a power is $\sigma_a^2=10.0$.

A.2. Decorrelator ***R*** is implemented by two decoupled pure recursive whitening filters. These two filters work in such way that one of them processes even samples, while the other processes odd samples, Fig 2. After switching the cascade into DFE the coefficients of one of two filters are translated into the feedback part of DFE.

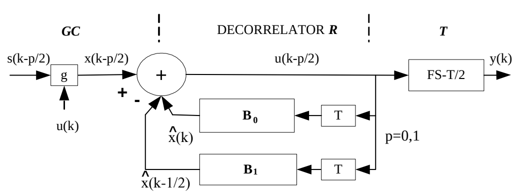

Fig. 2. Soft DFE: linear cascade ***GC-R-T*** for T/2-FS

For filter p=0 the input is $x(t)$ and the output is

$$u(t)=x(k)-\hat{x}(k) \qquad (3)$$

$$\hat{x}(k)=\sum_{i=1}^{N}b_iu(k-i)=B_0(k-1)^TU_N(k-1) \qquad (4)$$

$$U_N(k-1)=[u(k-1),...u(k-N)]^T \qquad (5)$$

$$B_0=[b_1,...b_{N-1},b_N]^T\,. \qquad (6)$$

The adaptive procedure for ***R*** is derived as a special case of the soft feedback model based on JEM criterion and a nonlinear monotone function $g(x)=\alpha\cdot\tanh(\beta\cdot x)$ [4]. Using the first two terms of the Taylor series expansion of g(x) we have obtained a new JEM type decorrelation algorithm [7]. This stochastic gradient algorithm for p=0 whitening filter is

$$B_0\left(k\right)=B_0\left(k-1\right)-\mu_Wu\left(k\right)\cdot\left[1-\beta_1\left|u(k)\right|^2\right]\cdot U_N^*\left(k-1\right) \qquad (7)$$

where $\mu_W=\mu\alpha^2\beta^2$ is a step size, and $\beta_1=\beta^2/3$ is a positive constant. The symbol * denotes complex conjugation. Note that the smoothness of function g(x) depends on parameter β, hence we can use β_1 to vary characteristics of algorithm (7).

A.3. Transversal filter ***T*** is Godard's equalizer

The input is $u\left(k-p/2\right)$, p=0,1 and the output is

$$y(k)=\sum_{i=0}^{L}c_iu(k-i/2)=C(k-1)^TU_{L+1}(k) \qquad (8)$$

$$C=\left[c_{0,}c_1,...c_L\right]^T \qquad (9)$$

$$U_{L+1}=\left[u(k),u(k-1/2),...u(k-L)\right]^T \qquad (10)$$

The Godard's algorithm minimizes dispersion of the output signal with respect to constant R_2 that depends on signal constellation

$$C(k)=C(k-1)+\mu_Gy(k)\left(\left|y(k)\right|^2-R_2\right)U_{L+1}^*(k) \qquad (11)$$

where μ_G is a step size. For the 16-QAM and 32-QAM schemes $R_2=13.2$.

B. JEM-DFE mode is introduced as a transition from the acquisition mode to the tracking one. It is based on JEM type algorithm and the corresponding scheme presented in Fig. 3. It should be emphasized that this scheme is a simplification of the original JEM soft feedback where the soft decision device g(x) has been replaced by slicer [see Fig. 1. in 4]. In our opinion, this approximation is not critical for Soft DFE because the most difficult task of blind equalization is already carried out by ***LE***.

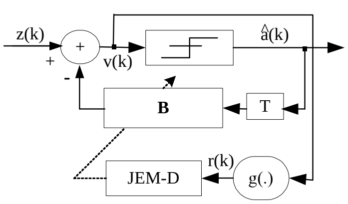

Fig.3. Soft DFE: JEM-DFE scheme

The input to decision device is given by

$$v(k)=z(k)-B(k-1)^T\hat{A}(k) \qquad (12)$$

$$z(k)=\left[C(k-1)^TU(k)\right]e^{-j\theta(k-1)} \qquad (13)$$

$$\hat{A}(k)=\left[\hat{a}(k-1),\hat{a}(k-2),...\hat{a}(k-L)\right]^T \qquad (14)$$

where $\theta(k)$ is output of the second-order carrier phase tracking loop.

The JEM type stochastic gradient algorithm for modified soft decision feedback is

$$B\left(k\right)=B\left(k-1\right)-\mu_Dv\left(k\right)\cdot\left[1-\beta_2\left|v(k)\right|^2\right]\cdot\hat{A}\left(k\right) \qquad (15)$$

where $\mu_D=\mu\alpha\beta$ is step size and $\beta_2=\beta^2/3$ is smoothing parameter for JEM-DFE.

C. The control of running modes is accomplished by performance monitoring procedure. It is estimator M of the output MSE [3], [7]. Since the true data are unknown, MSE is estimated using available data symbol estimates:

R_2 for acquisition mode (slicer is not turn on yet) and the slicer output $\hat{a}(k)$ for decision-directed mode

$$M_{AC}(k) = \lambda \cdot M_{AC}(k-1) + (1-\lambda)\left(\left|y(k)\right| - \sqrt{R_2}\right)^2 \quad (16)$$

$$M_{DD}(k) = \lambda \cdot M_{DD}(k-1) + (1-\lambda)\left(\left|v(k) - \hat{a}(k)\right|^2\right) \quad (17)$$

where λ is a so-called “forgetting factor” that is slightly less than one, e.g., λ=0.99 [3].

The monitor M makes control of switching operation modes according to the following rule:

$M_{AC}(k_0) \geq M_{TL-1}$, acquisition mode for $k > k_0$

$M_{DD}(k_1) < M_{TL-1}$, JEM-DFE mode for $k > k_1$

$M_{DD}(k_2) < M_{TL-2}$, tracking mode for $k > k_2$ (18)

where M_{TL-1} and M_{TL-2} are in advance defined threshold levels. Thus, the equalizer will switch from acquisition to JEM-DFE mode when M_{AC} becomes lower than M_{TL-1}. Obviously, this transition is the most critical phase of Hard/Soft DFE operation and the threshold M_{TL-1} has to be carefully selected. On the other hand, transition in the tracking mode, i.e., selection of the threshold M_{TL-2} is not critical because Soft DFE can stay in JEM-DFE mode. For example for 16-QAM and 32-QAM we have chosen M_{TL-1} =1.28 (+1 dB) and M_{TL-2} =0.16 (-8 dB).

III. SMOOTHING PARAMETERS SELECTION

The smoothing parameters β_1, and β_2 of algorithms (7) and (15), should be selected to provide a robust decorrelation of the signal with severe amplitude distortion, and a save (error propagation free) adaptation of the recursive part of DFE. Obviously, this optimization task does not lead to unique closed-form mathematical solution, hence we have applied the computer simulation tools to investigate this problem. For this purpose we have defined the kurtosis method with the aim to examine the effects of smoothing parameters to kurtosis of the different outputs of Soft DFE. Namely, it is well known that Godard’s and Shalvi-Weinstein equalizers maximize the kurtosis of the output sequence, which is equivalent to the zero-forcing (ZF) deconvolution of the noiseless system (channel + equalizer) with impulse response S. This optimization criterion based on Shalvi-Weinstein theorem [2] can be expressed as

$$F(S) = \frac{K(z(k))}{K(a(k))} = \left[\|S\|_4 / \|S\|_2\right]^4, \ \|S\|_q = \left[\sum_i |s_i|^q\right]^{1/q} \quad (19)$$

where $K(a(k))$ is the kurtosis of a zero-mean input sequence $\{a(k)\}$ and $K(z(k))$ is the kurtosis of the output sequence $\{z(k)\}$. It means that the Soft DFE in the acquisition mode constitutes a ZF equalizer and the above criterion will give a solution that inverts the linear cascade channel-***GC***-***R***. However, because a channel is unknown, we can recall a one-to-one correspondence between stationary points in the S and C domains [2], which

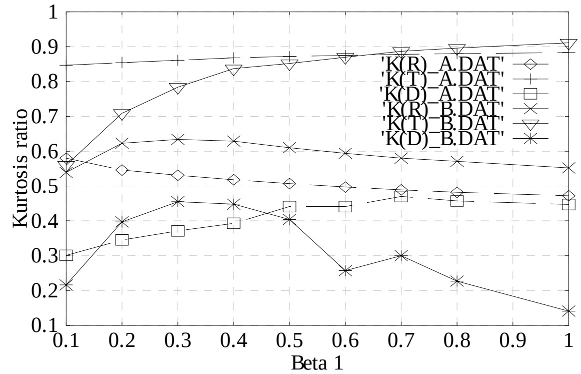

Fig. 4. Kurtosis ratio for outputs R, T, D: channel A (dash), channel B(solid)

TABLE 1: MSE (dB) AND THRESHOLD FAILES (TF-1,2) (%)

β_1	A:MSE	A: TF-1,2		B:MSE	B: TF-1,2	
0.1	-12.9	0	0	-7.1	21	29
0.2	-13.3	0	0	-9.7	8	12
0.3	-13.5	0	0	-11.1	3	10
0.4	-13.4	0	4	-11.7	0	12
0.5	-14.0	0	2	-11.7	0	18
0.6	-13.9	0	4	-8.2	0	48
0.7	-14.3	0	4	-9.6	0	38
0.8	-14.1	0	7	-7.9	0	52
1.0	-14.0	0	11	-5.4	0	70
Hard	-12.2	0	0	-4.5	20	57

suggests to evaluate $F(S)$ with respect to vector C calculating only the vector length $\|C\|$ of ***T***.

According to elaborated approach we have defined the following relations for calculating the normalized kurtosis ratio of the outputs of ***R***, ***T*** and recursive part of DFE (***D***)

$$K_R(u_s) \triangleq \left[\|B_0\|_4 / \|B_0\|_2\right]^4 \quad (20)$$

$$K_T(z_s) \triangleq \left[\|C\|_4 / \|C\|_2\right]^4 \quad (21)$$

$$K_D(\hat{a}_s) \triangleq \left[\|B\|_4 / \|B\|_2\right]^4 \quad (22)$$

where $u_S = u(k_S)$ and $z_S = z(k_S)$ are the outputs of ***R*** and ***T***, at the moment of reaching threshold MSE_{TL-1}, and $\hat{a}_s = \hat{a}(k_S)$ is the output of ***D*** at the moment of reaching threshold MSE_{TL-2}. The corresponding lengths of vector coefficients of ***R***, ***T*** and ***D*** are $\|B_0\|$, $\|C\|$ and $\|B\|$, respectively.

The described kurtosis method is used to examine the influence of parameter β_1 to acquisition and JEM-DFE modes of operation for the fixed value of β_2. The Monte Carlo tests running N_{RUN}=100 independent simulations, each of which lasting 30000 T intervals, are carried out for 16-QAM scheme and noiseless multipath channels A and B, which are presented in the next section. The obtained results are presented in Fig. 4 and Table 1.

Note, the kurtosis values $K_T(z_s)$ for the both channels show saturation for β_1>0.4. On the other hand, the kurtosis $K_{R'}(\hat{a}_s)$ reaches the maximum values for

$\beta_1 \cong 0.7$ and $\beta_1 \cong 0.4$ for channels A and B, respectively. These maximum kurtosis values are correlated with the corresponding minimum values of MSE that are measured at the end of the tests, Table 1. Besides, Table 1. presents results of unsuccessful passing of thresholds M_{TL-1} and M_{TL-2}, which in the case of more severe channel B indicate that the selected value of β_1 should provide the compromise between successful structure switching and minimum MSE. In the other words, the increasing of β_1 leads to better decorrelation in the acquisition mode but, on the other hand, too large values of β_1 cause the unacceptable bad MSE performance in the tracking mode.

IV. Performance Testing

In this section we present the results of testing two solutions of blind equalizers, Hard DFE and Soft DFE, carried out by a software simulator, which has been originally designed for ITU-T V.32 modem. The transfer function of applied transmitter and receiver filters follows a raised cosine with roll-off factor 0.12. The channels represent a three-ray multipath environment whose impulse response is given by

$$h(t) = e(t)W(t) + d_1 e(t-\tau_1)W(t-\tau_1) + d_2 e(t-\tau_2)W(t-\tau_2) \quad (23)$$

where e(t) is the basic pulse, W(t) is a rectangular window spanning [-16T,16T], d_i is attenuation factor of *i*th channel, and τ_i is propagation delay of *i*th path [4]. The multipath parameters d_i and τ_i take the following values for channels A and B: $d_1 \in \{0.9, 0.8\}$, $d_2 \in \{0.35, 0.40\}$, $\tau_1 \in \{3(T/4), 2(T/4)\}$, $\tau_2 \in \{2T, 2T\}$, respectively. The initial kurtosis values of the combination (transmitter filter +channel) for channels A and B (see Fig. 5.) are: $K_{A,p=0} = 0.442$, $K_{A,p=1} = 0.368$, $K_{B,p=0} = 0.421$, $K_{B,p=1} = 0.301$.

Both, Hard DFE and Soft DFE equalizers have N=6, L+1=42 taps in recursive and transversal parts, respectively. The centered reference tap, receiving the input stream p=1, has an initial value 3.1. Taking into account the previous analyses that addresses selection of the parameter β_1, we have estimated β_1=0.5 as a best compromise between equalization requests in blind and decision-directed modes of operation.

The convergence characteristics of DFE blind equalizers for 16-QAM signal and the input signal-noise ration SNR=25 dB are presented in Fig. 6. These results show that Soft DFE acquires the significantly better minimum MSE performance in comparison to the present Hard DFE solution.

V. Conclusion

The proposed Soft DFE is a performance efficient solution of blind DFE based on the "adaptive" FS structure. Its most important characteristic is the application of JEM-type algorithms for the critical phases of operations: 1) during the blind acquisition and 2) immediately after the structure switching. The contribution of a new JEM decorrelator to blind acquisition has been examined by extensive simulations, which have shown a better convergence rate and robustness in comparison with the presently used whitening filter.

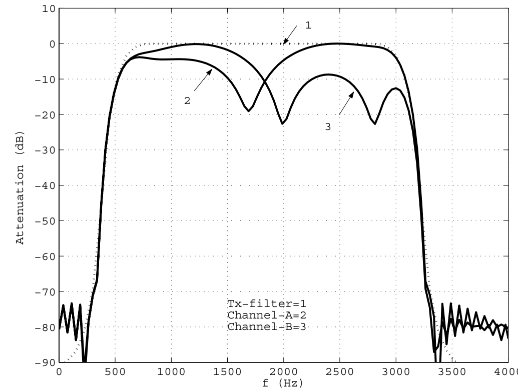

Fig. 5. Attenuation characteristics of channels A and B

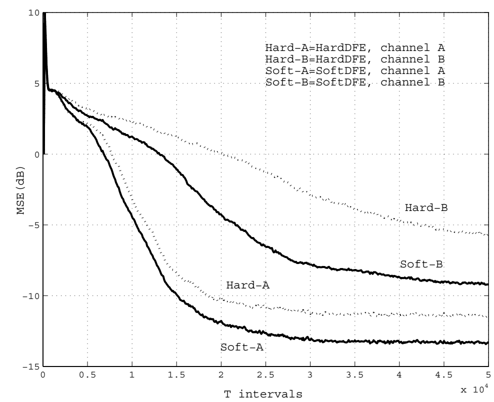

Fig. 6. Minimum MSE comparison of two solutions for channels A and B: 16-QAM, SNR=25 dB, β_1=0.5,β_2=5

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